IMPROVED DETERMINATION OF THE b-QUARK MASS AT THE Z PEAK [a]

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Next-to-leading order calculations of heavy quark three-jet production in e^+e^- annihilation are reviewed. Their application for the measurement of the b-quark mass at LEP/SLC and to the test of the flavour independence of the strong coupling constant are discussed. Prospects for future improvements are studied.

1 Introduction

Effects of the bottom-quark mass, m_b, were already noticed in the early tests [1] of the flavour independence of the strong coupling constant, α_s, in e^+e^--annihilation at the Z-peak. Motivated by the remarkable sensitivity of the three-jet observables to the value of the quark mass, the possibility of the determination of m_b at LEP, assuming universality of the strong interactions, was considered [2]. This question was analyzed in detail in [2], where the necessity of the next-to-leading order (NLO) calculation for the measurements of m_b was also emphasized.

The NLO calculation for the process $e^+e^- \to 3jets$, with complete quark mass effects, has been performed independently by three groups [3,4,5]. These predictions are in agreement with each other and were successfully used in the measurements of the b-quark mass far above threshold [6,8] and in the precision tests of the universality of the strong interaction [6,7,8] at the Z-pole. In this talk we make a short review of such calculations. Furthermore, prospects for future improvements are discussed.

It is surprising that at high energies the bottom-quark mass could be relevant, since it appears screened by the center of mass energy, $m_b^2/m_Z^2 \simeq 10^{-3}$ at LEP. Nevertheless, when more exclusive processes than a total cross section

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are considered, like a n-jet cross section, mass effects can be enhanced as $m_b^2/m_Z^2/y_c$, where y_c is the parameter that defines the jet multiplicity.

Since quarks are *not free* particles and, therefore, their mass can be considered like a coupling constant, one has the freedom to use different quark mass definitions, e.g. the perturbative pole mass M_b or the $\overline{MS}$ scheme running mass $m_b(\mu)$. Physics should be independent of it but at a fixed order in perturbation theory there is a significant dependence on which mass definition is used, as well as on the renormalization scale μ. The inclusion of higher orders to reduce these two uncertainties, due to mass definition and μ scale, is mandatory for an accurate description of mass effects.

2 Three jet observables and the measurement of m_b

The observable proposed some time ago to measure the bottom-quark mass at the Z-resonance was the ratio [2]

$$R_3^{bd} \equiv \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^d(y_c)/\Gamma^d} \ , \tag{1}$$

where Γ_{3j}^q and Γ^q are the three-jet and the total decay widths of the Z-boson into a quark pair of flavour q in a given jet-clustering algorithm. More precisely, the measured quantity is

$$R_3^{b\ell} \equiv \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^\ell(y_c)/\Gamma^\ell} = 1+\frac{\alpha_s(\mu)}{\pi}a_0(y_c)+r_b\left(b_0(r_b,y_c)+\frac{\alpha_s(\mu)}{\pi}\,b_1(r_b,y_c)\right) \ , \tag{2}$$

where now the sum of the contributions of the three light flavours $\ell = u, d, s$ is included in the denominator. The R_3^{bd} and the $R_3^{b\ell}$ observables differ only by the function a_0 (which is zero for R_3^{bd}). This contribution originates from the triangle diagrams [11]. It is numerically very small (0.002 for the Durham jet algorithm) and almost independent of the b-quark mass. The b_0 and b_1 functions give respectively the leading-order (LO) and NLO mass corrections, once the leading dependence on $r_b = M_b^2/m_Z^2$, where M_b is the bottom-quark pole mass, has been factorized out.

Ratios of differential two-jet rates, where the two-jet width Γ_{2j} is calculated from the three- and the four-jet fractions through the identity $\Gamma_{2j} = \Gamma - \Gamma_{3j} - \Gamma_{4j}$, have been studied in [12,4]. Ratios of event shape distributions have also been considered [5].

Using the known relationship between the pole mass and the $\overline{MS}$ scheme running mass,

$$M_b^2 = m_b^2(\mu)\left[1+\frac{2\alpha_s(\mu)}{\pi}\left(\frac{4}{3}-\log\frac{m_b^2}{\mu^2}\right)\right] \ , \tag{3}$$

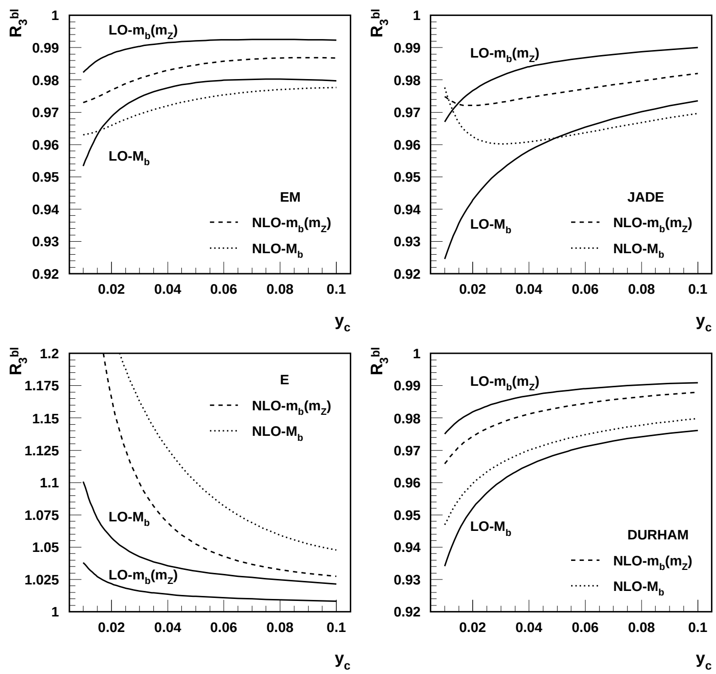

Figure 1: The observable $R_3^{b\ell}$ as a function of y_c at the NLO for the four algorithms considered. The dotted lines give the observable computed at the NLO in terms of the pole mass $M_b = 4.6$ GeV while the dashed lines are obtained when it is written in terms of the running mass $m_b(m_Z) = 2.83$ GeV. In both cases the renormalization scale is fixed at $\mu = m_Z$, and $\alpha_s(m_Z) = 0.118$. For comparison we also plot in solid lines the LO results for $M_b = 4.6$ GeV (LO-M_b) and $m_b(m_Z) = 2.83$ GeV (LO-$m_b(m_Z)$).

we can re-express Eq. (2) in terms of the running mass $m_b(\mu)$. Then, keeping only terms of order $\mathcal{O}(\alpha_s)$ we obtain

$$R_3^{b\ell} = 1 + \frac{\alpha_s(\mu)}{\pi}\, a_0(y_c) + \bar{r}_b(\mu) \left(b_0(\bar{r}_b, y_c) + \frac{\alpha_s(\mu)}{\pi}\, \bar{b}_1(\bar{r}_b, y_c, \mu) \right) \ , \qquad (4)$$

where $\bar{r}_b(\mu) = m_b^2(\mu)/m_Z^2$ and $\bar{b}_1 = b_1 + 2b_0(4/3 - \log r_b + \log(\mu^2/m_Z^2))$. Although at the perturbative level both expressions, Eq. (2) and Eq. (4), are equivalent they give different answers since different higher order contributions have been neglected. The spread of the results gives an estimate of the size of higher order corrections.

In fig. 1 we present our results for $R_3^{b\ell}$ in the four clustering algorithms: EM [b], Jade, E and Durham. For all the algorithms we plot the NLO results written either in terms of [10] the pole mass, $M_b = 4.6$ GeV, or in terms of the running mass at m_Z, $m_b(m_Z) = 2.83$ GeV. The renormalization scale is fixed to $\mu = m_Z$ and $\alpha_s(m_Z) = 0.118$. For comparison we also show $R_3^{b\ell}$ at LO when the value of the pole mass, M_b, or the running mass at m_Z, $m_b(m_Z)$, is used for the quark mass.

Note the different behaviour of the different algorithms. In particular the E algorithm. As already discussed in [2], in this algorithm the shift in the resolution parameter produced by the quark mass makes the mass corrections positive while by kinematical arguments one would expect a negative effect, since massive quarks radiate less gluons than massless quarks. Furthermore, the NLO corrections are very large in the E algorithm and strongly dependent on y_c. All this probably indicates that it is difficult to give an accurate QCD prediction for it. For the Jade algorithm the NLO correction written in terms of the pole mass starts to be large for $y_c \leq 0.02$. Note, however that the NLO correction written in terms of the running mass is still kept in a reasonable range in this region. Durham, in contrast, is the algorithm that presents a better behaviour for relatively low values of y_c while keeping NLO corrections in a reasonable range.

The theoretical predictions for the observables studied contain a residual dependence on the renormalization scale μ. To give an idea of the uncertainties introduced by this we plot in fig. 2a the observable $R_3^{b\ell}$ as a function of μ for a fixed value of y_c. Here we only present plots for the Durham algorithm, the one with the better behaviour. We use the following one-loop evolution equations

$$a(\mu) = \frac{a(m_Z)}{K} , \qquad m_b(\mu) = m_b(m_Z)\, K^{-\gamma_0/\beta_0} , \tag{5}$$

where $a(\mu) = \alpha_s(\mu)/\pi$, $K = 1 + a(m_Z)\beta_0 \log(\mu^2/m_Z^2)$ with $\beta_0 = (11 - 2/3N_F)/4$, $\gamma_0 = 1$ and $N_F = 5$ the number of active flavours, to connect the running parameters at different scales.

Conversely, for a given value of $R_3^{b\ell}$ we can solve Eq. (2) (or Eq. (4)) with respect to the quark mass. The result, shown in fig. 2b for $R_3^{b\ell}(y_c = 0.02) = 0.973$, depends on which equation was used and has a residual dependence on the renormalization scale μ. The curves in fig. 2b are obtained in the following way: first from Eq. (4) we directly obtain for an arbitrary value of μ between m_Z and $m_Z/10$ a value for the bottom-quark running mass at that scale, $m_b(\mu)$, and then using Eq. (5) we get a value for it at the Z-scale, $m_b(m_Z)$. Second, using Eq. (2) we extract, also for an arbitrary value of μ between m_Z

[b] A modification of the standard Jade scheme, convenient for massive parton calculations [2].

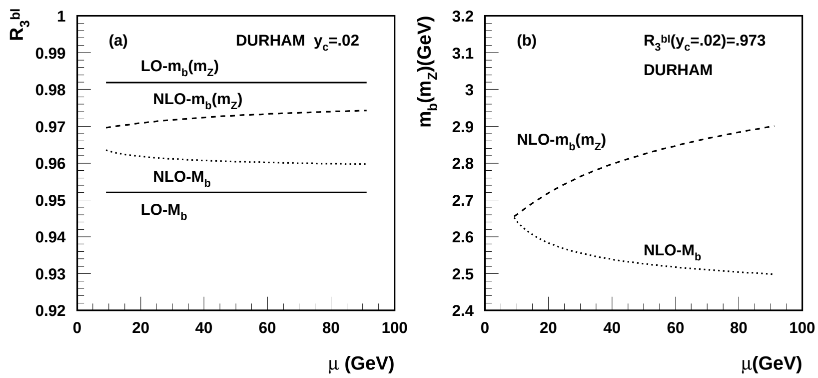

Figure 2: Durham algorithm: (a) Renormalization scale dependence of the NLO predictions written in terms of either the running mass, NLO-$m_b(m_Z)$, or the pole mass, NLO-M_b, for the $R_3^{b\ell}$ ratio at a fixed value of y_c. (b) Predicted value of $m_b(m_Z)$ for a fixed value of $R_3^{b\ell}$ using either the running expression, NLO-$m_b(m_Z)$, or the pole expression, NLO-M_b, as a function of the scale μ.

and $m_Z/10$, a value for the pole mass, M_b. Then we use Eq. (3) at $\mu = M_b$ and again Eq. (5) to perform the evolution from $\mu = M_b$ to $\mu = m_Z$ and finally get a value for $m_b(m_Z)$. The two procedures give a different answer since different higher orders have been neglected in the intermediate steeps. The maximum spread of the two results, in this case of the order of ± 200 MeV, can be interpreted as an estimate of the size of higher order corrections, that is, of the theoretical error in the extraction of $m_b(m_Z)$ from the experimental measurement of $R_3^{b\ell}$.

These calculations were used by the DELPHI Coll. [6] to extract $m_b(m_Z)$ from the experimental measurement of $R_3^{b\ell}$, see fig. 3, and the result was interpreted as the first experimental evidence (at 2-3 sigmas) for the *running* of a fermion mass since the data are better described by the NLO-$m_b(m_Z)$ curve. Also recently the SLD Coll. [8] has presented results for $m_b(m_Z)$. The SLD analysis is compatible with the DELPHI measurement. Nevertheless, the central values of $m_b(m_Z)$ obtained from different clustering algorithms are scattered in the range $\Delta m_b(m_Z) = \pm 0.49$ GeV. This is probably due to the fact that E-like algorithms, that are mainly used in this analysis, have huge NLO corrections thus making accurate QCD predictions difficult.

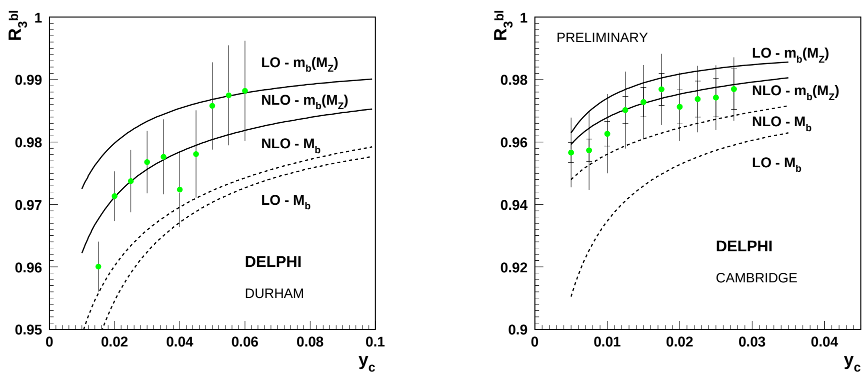

Figure 3: Experimental measurement of $R_3^{b\ell}$ by the DELPHI Coll. in the Durham algorithm and preliminary results in the Cambridge algorithm compared with our NLO calculation written in terms of the running mass at the Z peak, NLO-$m_b(m_Z)$, or in terms of the pole mass, NLO-M_b. The LO predictions are also plotted.

3 Tests of the flavour independence of the strong interaction

Assuming a given b-quark mass the NLO calculation of heavy quark three-jet production cross section can be used to perform an improved test of the flavour independence of the strong coupling constant. Such analysis was done for the first time by the DELPHI Coll. in [6] by using the $R_3^{b\ell}$ observable defined in the Durham algorithm. Recently, the OPAL Coll. [7] has presented a similar analysis by using different ratios of event shapes distributions: D_2, $1-T$, M_H, B_T, B_W and C. Instead, SLD has presented [8] results by analyzing the R_3^{bl} ratio in the E, E0, P, P0, Durham and Geneva algorithms. All these results are consistent with unity. *No flavour dependence* has been observed. Furthermore, the inclusion of mass effects is mandatory to achieve such agreement.

4 Improving the b-quark mass measurements: the Cambridge algorithm

The Cambridge algorithm has been introduced very recently [9] in order to reduce the formation of spurious jets formed with low transverse momentum particles that appear in the Durham algorithm at low y_c. Therefore, compared to Durham, it allows to test smaller values of y_c while still keeping NLO corrections relatively small. Both algorithms are defined by the same recombination procedure and the same test variable

$$y_{ij} = 2min(E_i^2, E_j^2)(1 - \cos\theta_{ij})/s \ , \tag{6}$$

where E_i and E_j denote the energies of particles i and j and θ_{ij} is the angle between their momenta. The new ingredient of the Cambridge algorithm is the so called ordering variable

$$v_{ij} = 2(1 - \cos\theta_{ij}) \ . \tag{7}$$

In the Cambridge algorithm one first finds the minimal v_{ij} and then tests y_{ij}. If $y_{ij} < y_c$, the i and j particles are recombined into a new pseudoparticle of momentum $p_k = p_i + p_j$ but if $y_{ij} > y_c$, the softer particle is resolved as a jet. The net effect of the new definition is that NLO corrections to the three-jet fraction become smaller.

In fig. 3 we present the preliminary results from the DELPHI Coll.[13] for the $R_3^{b\ell}$ ratio defined in the Cambridge algorithm and compare it with our NLO calculation[12] written in terms of the running mass at the Z peak, NLO-$m_b(m_Z)$, or in terms of the pole mass, NLO-M_b, for $\mu = m_Z$ and $\alpha_s(m_Z) = 0.118$. As in Durham, the NLO-$m_b(m_Z)$ gives the best agreement. Furthermore, data are still compatible with the LO-$m_b(m_Z)$ showing that the bulk of higher order corrections is described by the *running of the b-quark mass*. In contrast, although data could also be well described by the NLO-M_b curve, the NLO corrections become large when the pole mass parametrization is used.

The studies of the NLO-$m_b(m_Z)$ curve show[12] that it is remarkably stable with respect to the variation of the scale μ. For the range $m_Z/10 < \mu < m_Z$ the estimate of the error in the extracted $m_b(m_Z)$ is reduced to ± 50 MeV in the Cambridge scheme, with respect to ± 200 MeV for Durham (± 125 MeV if only NLO-$m_b(m_Z)$ is considered). In contrast, when the NLO-M_b parametrization is used we get ± 240 MeV but strongly dependent on the lower μ used.

5 Conclusions

In the last few years an important progress was done in the description of the Z-boson decay into three-jets with massive quarks. Next-to-leading order calculations have been done by three groups and have been successfully used in the analysis of the LEP and SLC data where mass effects have been clearly seen. Further studies of different observables and different jet-algorithms are oriented on the reduction of the theoretical uncertainty. One good candidate might be the Cambridge jet-algorithm, where the NLO corrections are particularly small and where the predictions in terms of the running mass, $m_b(m_Z)$ are particularly stable with respect to the variation of the renormalization scale.

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